Mathematical models of hysteresis phenomena

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Hysteresis may be defined as a *rate-independent memory effect,* and occurs in several phenomena in physics and other disciplines. It was represented via continuous *hysteresis operators* in the 1970s by Krasnosel'skiĭ and co-workers [KrPo]. Some models of hysteresis in elasto-plasticity may be represented via variational inequalities, and may thus be dealt within the framework of convex analysis. Others require a different approach.

Partial differential equations with hysteresis have been considered since the early 1980s. They include quasilinear parabolic and hyperbolic equations of the form

$$\frac{\partial}{\partial t}[u + \mathcal{F}(u)] + Au = f, \tag{1}$$

$$\frac{\partial^2}{\partial t^2}[u + \mathcal{F}(u)] + Au = f. \tag{2}$$

Here $\mathcal{F}$ denotes a continuous hysteresis operator, and A is a linear elliptic operator. A suitable monotonicity-type condition is assumed for $\mathcal{F}$. The Cauchy problem for (1) has one and only one solution [Vi1,Vi3].

A different approach is in order in case of discontinuous hysteresis, e.g. for the simple *(delayed) relay operator,* which is characterized by a rectangular hysteresis loop. Although this is single relation, it can be represented as a system of two variational inequalities. Existence and uniqueness of a weak solution for the problem (1) was proved also in this case.

The Cauchy problem for the hyperbolic equation (2) also has a weak solution [Vi2, Vi3]. This rests upon the regularization effect of hysteresis.

Details may be found in the monograph [Vi1] and the survey [Vi3]. A different approach is addressed in [MiRo].

References

[KrPo] M.A. Krasnosel'skiĭ, A.V. Pokrovskiĭ: *Systems with Hysteresis.* Springer, Berlin 1989 (Russian ed. Nauka, Moscow 1983)

[MiRo] A. Mielke, T. Roubíček: *Rate-independent systems. Theory and application.* Applied Mathematical Sciences, vol. 193. Springer, New York, 2015

[Vi1] A.V. *Differential models of hysteresis.* Applied Mathematical Sciences, vol. 111. Springer, Berlin (1994)

[Vi2] A.V. *Quasi-linear hyperbolic equations with hysteresis.* Ann. Inst. H. Poincaré. Analyse non linéaire, 19 (2002), 451–476

[Vi3] A.V. *Mathematical models of hysteresis.* In: The Science of Hysteresis (G. Bertotti, I. Mayergoyz, eds.) Elsevier (2006), chap. 1, pp. 1–123